Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric then what is x equal to? **[1]**

   a) 5 b) 2

   c) 3 d) -1

2. If A and B are square matrices of order 3, such that Det.A = –1, Det.B =3 then, the determinant of 3AB is equal to **[1]**

   a) –9 b) –27

   c) -81 d) 81

3. The inverse of the matrix $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$ is **[1]**

   a) $\begin{vmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{vmatrix}$ b) $\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix}$

   c) $\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}$ d) $\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$

4. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ $\frac{dy}{dx} = ?$ **[1]**

   a) tan θ b) a cot θ

   c) a tan θ d) cot θ

5. Equation of a line passing through point (1, 1, 1) and parallel to z-axis is **[1]**

   a) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$ b) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

   c) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ d) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$

6. The general solution of the differential equation $e^x dy + (y e^x + 2x) dx = 0$ is [1]

a) $x e^y + y^2 = C$

b) $y e^y + x^2 = C$

c) $x e^y + x^2 = C$

d) $y e^x + x^2 = C$

7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).Let F = 4x + 6y be the objective function. The Minimum value of F occurs at [1]

a) any point on the line segment joining the points (0, 2) and (3, 0).

b) (0, 2) only

c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only

d) (3, 0) only

8. The value of $\sin^{-1}\left(\cos \frac{3\pi}{5}\right)$ is [1]

a) $\frac{-3\pi}{5}$

b) $\frac{-\pi}{10}$

c) $\frac{\pi}{10}$

d) $\frac{3\pi}{5}$

9. $\int \frac{dx}{\sqrt{x-x^2}} = ?$ [1]

a) $\sin^{-1}(2x + 1) + C$

b) $\sin^{-1}(2x - 1) + C$

c) $\sin^{-1}(x + 1) + C$

d) $\sin^{-1}(x - 1) + C$

10. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A^4 = [1]

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

11. Maximize Z = 100x + 120y , subject to constraints $2x + 3y \leq 30$, $3x + y \leq 17$, $x \geq 0$, $y \geq 0$. [1]

a) 1200

b) 1260

c) 1280

d) 1300

12. What is the vector perpendicular to both the vectors $\hat{i} - \hat{j}$ and $\hat{i}$? [1]

a) $\hat{j}$

b) $\hat{k}$

c) $-\hat{j}$

d) $\hat{i}$

13. The value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = 3A - B then the values of A and B are: [1]

a) A = 2abc, B = a + b + c

b) A = 3abc, B = a + b + c

c) A = 0, $B = a^2 + b^2 + c^2$

d) A = abc, $B = a^3 + b^3 + c^3$

14. If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to: [1]

a) $\frac{1}{2}$

b) $\frac{2}{3}$

c) $\frac{1}{3}$

d) $\frac{1}{6}$

15. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is [1]

a) Not defined | b) 1
c) 2 | d) 3

16. The position vectors of three consecutive vertices of a parallelogram ABCD are A$(4\hat{i}+2\hat{j}-6\hat{k})$, B $(5\hat{i}-3\hat{j}+\hat{k})$ and C$(12\hat{i}+4\hat{j}+5\hat{k})$. The position vector of D is given by **[1]**

a) $-11\hat{i}-9\hat{j}+2\hat{k}$
b) $-3\hat{i}-5\hat{j}-10\hat{k}$
c) $11\hat{i}+9\hat{j}-2\hat{k}$
d) $21\hat{i}+3\hat{j}$

17. The derivative of sin x w.r.t. cos x is **[1]**

a) cosec x
b) tan x
c) -cot x
d) cot x

18. The lines l_1 and l_2 intersect. The shortest distance between them is **[1]**

a) zero
b) positive
c) infinity
d) negative

19. **Assertion (A):** A particle moving in a straight line covers a distance of x cm in t second, where $x = t^3 + 3t^2 - 6t + 18$. The velocity of particle at the end of 3 seconds is 39 cm/s. **[1]**

Reason (R): Velocity of the particle at the end of 3 seconds is $\frac{dx}{dt}$ at t = 3.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The number of onto functions from a set P containing 5 elements to a set Q containing 2 elements is 30. **[1]**

Reason (R): Number of onto functions from a set containing m elements to a set containing n elements is n^m.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Write the interval for the principal value of function and draw its graph: $\sec^{-1} x$. **[2]**

OR

Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

22. Find the intervals of function $f(x) = (2x^3 + 9x^2 + 12x + 15)$ is **[2]**

a. increasing
b. decreasing.

23. A man 1.6m tall walks at the rate of 0.3 m/s away from a street light is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening? **[2]**

OR

For the function $f(x) = 2x^3 - 3x^2 - 12x + 4$, find the points of local maxima and minima.

24. Find $\int \frac{dx}{2\sin^2 x+5\cos^2 x}$ **[2]**

[2]

25. If $A = \begin{bmatrix} \cos\theta & \cos\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then fin the value of k.

Section C

26. Evaluate the integral: $\int x^2 \tan^{-1} x dx$ **[3]**

27. In a game a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amounts he wins/loses. **[3]**

28. Prove that $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log\frac{2}{3}$ **[3]**

OR

Evaluate: $\int \frac{1}{\cos 2x + 3\sin^2 x} dx$

29. Solve differential equation: $(1 + y + x^2y)dx + (x + x^3)dy = 0$ **[3]**

OR

Find the particular solution of the differential equation $\frac{dy}{dx}$ + 2y tan x = sin x, given that y = 0 when x = $\frac{\pi}{3}$.

30. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are three vectors such that $|\vec{a}| = 5$, $|\vec{b}| = 12$ and $|\vec{c}| = 13$, and $\vec{a} + \vec{b} + \vec{c} = \vec{o}$ find the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ **[3]**

OR

Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and other is perpendicular to $\vec{b}$.

31. If x = $\tan\left(\frac{1}{a}\log y\right)$, then show that $\left(1 + x^2\right)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$. **[3]**

Section D

32. Find the area between the curves y = x and y = x^2 **[5]**

33. Show the relation R in the set A = {x ∈ Z : 0 ≤ x ≤12}, given by R = {(a, b) : a = b}, is an equivalence relation. Find the set of all elements related to 1 in each case. **[5]**

OR

Let R be relation defined on the set of natural number N as follows:

R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

34. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. **[5]**

35. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$. **[5]**

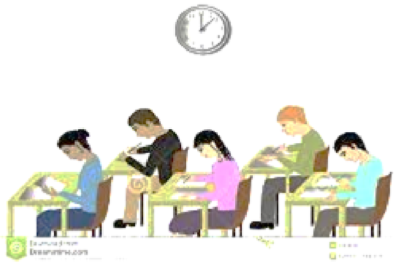
OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.



i. Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics? (1)

ii. Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics? (1)

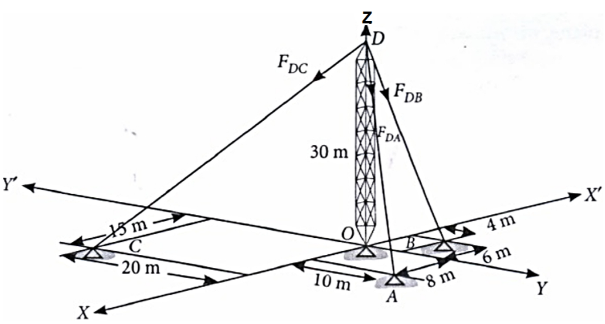
iii. Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics? (2)

OR

Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Consider the following diagram, where the forces in the cable are given.



i. What is the equation of the line along cable AD? (1)

ii. What is length of cable DC? (1)

iii. Find vector DB (2)

OR

What is sum of vectors along the cable? (2)

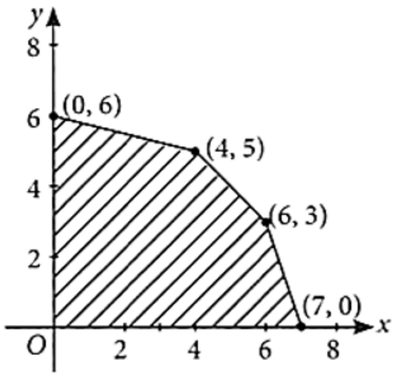
38. **Read the following text carefully and answer the questions that follow:** **[4]**

Linear programming is a method for finding the optimal values (maximum or minimum) of quantities subject to the constraints when a relationship is expressed as linear equations or inequations.

i. At which points is the optimal value of the objective function attained? (1)

ii. What does the graph of the inequality $3x + 4y < 12$ look like? (1)

iii. Where does the maximum of the objective function $Z = 2x + 5y$ occur in relation to the feasible region shown in the figure for the given LPP? (2)



OR

What are the conditions on the positive values of p and q that ensure the maximum of the objective function Z = px + qy occurs at both the corner points (15, 15) and (0, 20) of the feasible region determined by the given system of linear constraints? (2)

Solution

Section A

1. **(a)** 5

Explanation:

$\because A = A' \Rightarrow \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix} = \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$

$\Rightarrow$ 2x - 3 = x + 2 $\Rightarrow$ x = 5

2.

(c) -81

Explanation:

$|3AB| = 3^3|A||B| = 27(-1)(3) = -81$

3. **(a)** $\begin{vmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{vmatrix}$

Explanation:

A = $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$, adj A = $\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix}$

|A| = 8 + 3 = 11

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{11}\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix} = \begin{vmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{vmatrix}$

4. **(a)** tan θ

Explanation:

$x = a(\cos\theta + \theta\sin\theta)$,we get

$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$

$y = a(\sin\theta - \theta\cos\theta)$,we get

$\therefore \frac{\mathrm{dy}}{\mathrm{d}\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = \mathrm{a}\cos\theta - \mathrm{a}\cos\theta + \theta\mathrm{a}\sin\theta$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{d}\theta} = \mathrm{a}\theta\sin\theta$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{dy}}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{dx}}$

$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$

$\Rightarrow \frac{dy}{dx} = \tan\theta$

5.

(b) $\frac{\mathrm{x}-1}{0} = \frac{\mathrm{y}-1}{0} = \frac{\mathrm{z}-1}{1}$

Explanation:

$\frac{\mathrm{x}-1}{0} = \frac{\mathrm{y}-1}{0} = \frac{\mathrm{z}-1}{1}$

6.

(d) y $e^x + x^2$ = C

Explanation:

It is given that e^xdy + (ye^x + 2x) dx = 0

$\Rightarrow \mathrm{e}^{\mathrm{x}}\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{ye}^{\mathrm{x}} + 2\mathrm{x} = 0$

$\Rightarrow \frac{dy}{dx} + y = -2xe^{-x}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = 1 and Q = -2xe^{-x})

Now, I.F = $\mathrm{e}^{\int \mathrm{pdx}} = \mathrm{e}^{\int \mathrm{dx}} = \mathrm{e}^{\mathrm{x}}$

Thus, the solution of the given differential equation is given by the relation:

$y(I.F.) = \int (Q \times I.F.)dx + C$

$\Rightarrow ye^x = \int (-2xe^{-x} \cdot e^x)\,dx + C$

$\Rightarrow ye^x = -\int 2xdx + C$

$\Rightarrow ye^x = -x^2 + C$

$\Rightarrow ye^x + x^2 = C$

7. **(a)** any point on the line segment joining the points (0, 2) and (3, 0).

Explanation:

Here the objective function is given by : F = 4x +6y.

Corner points	**Corresponding value of F = 4x + 6y**
(0, 2)	12 $\leftarrow$ Minimum
(3, 0)	12 $\leftarrow$ Minimum
(6, 0)	24
(6, 8)	72
(0, 5)	30

Hence it is clear that the minimum value occurs at any point on the line joining the points (0,2) and (3,0)

8.

(b) $\frac{-\pi}{10}$

Explanation:

$\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right)$

$= \sin^{-1}\sin\left(\frac{5\pi - 6\pi}{10}\right)$

$= \sin^{-1}\sin\left(-\frac{\pi}{10}\right)$

$= -\frac{\pi}{10} \epsilon \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

9.

(b) $\sin^{-1}(2x - 1) + C$

Explanation:

The given integral is $\int \frac{dx}{\sqrt{x - x^2}} = ?$

$(x - x^2) = \frac{1}{4} - \left(x^2 - x + \frac{1}{4}\right) = \left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2$

$\therefore I = \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}} = \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - t^2}} = \sin^{-1}\frac{t}{(1/2)} + C$

$= \sin^{-1}2t + C = \sin^{-1}2\left(x - \frac{1}{2}\right) + C = \sin^{-1}(2x - 1) + C$

10.

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explanation:

In the given question the given matrix is an Identity matrix, and I^n = I.I.I .... .I(n times) = I.

11.

(b) 1260

Explanation:

We have , Maximize Z = 100x + 120y , subject to constraints 2x + 3y ≤ 30, 3x + y ≤ 17, x ≥ 0, y ≥ 0.

Corner points	Z = 100x +120 y
P(0 , 0)	0
Q(3 , 8)	1260…………………(Max.)
R(0, 10)	1200
S(17/3 , 0)	1700/3

Hence the maximum value is 1260

12.

(b) $\hat{k}$

Explanation:

The vector perpendicular to both the vectors $(\hat{i}-\hat{j})$
and $\hat{i} = (\hat{i}-\hat{j}) \times \hat{i}$ = $\hat{i} \times \hat{i} - \hat{j} \times \hat{i}$
= 0 + $\hat{i} \times \hat{j} = \hat{k}$

13.

(d) A = abc, B = $a^3 + b^3 + c^3$

Explanation:

$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = $a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

= $abc - a^3 - b^3 + abc + abc - c^3$

= $3abc - (a^3 + b^3 + c^3)$

$\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = $3abc - (a^3 + b^3 + c^3)$

$\therefore$ A = abc, B = $a^3 + b^3 + c^3$

14. **(a)** $\frac{1}{2}$

Explanation:

$\frac{1}{2}$

15. **(a)** Not defined

Explanation:

It is given that equation is $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$

The given differential equation is not a polynomial equation in its derivative

Therefore, its degree is not defined.

16.

(c) $11\hat{i} + 9\hat{j} - 2\hat{k}$

Explanation:

$11\hat{i} + 9\hat{j} - 2\hat{k}$

17.

(c) -cot x

Explanation:

-cot x

18. **(a)** zero

Explanation:

Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

We have,

x = $t^3 + 3t^2 - 6t + 18$

Velocity, $v = \frac{dx}{dt} = 3t^2 + 6t - 6$

Thus, velocity of the particle at the end of 3 seconds is

$\left(\frac{dx}{dt}\right)_{t=3} = 3(3)^2 + 6(3) - 6$

= 27 + 18 - 6 = 39 cm/s

20.

(c) A is true but R is false.

Explanation:

A is true but R is false.

Set P contains 5 elements and set Q contains 2 elements.

Let the number of elements in P be m i.e. m=5

and the number of elements in Q be n i.e. n=2.

Number of onto function = $\sum_{r=0}^{n-1} {}^{n}c_r(-1)^r(n-r)^m$

$= \sum_{r=0}^{1} {}^{2}c_r(-1)^r(2-r)^5$

$= {}^{2}c_0(-1)^0(2-0)^5 + {}^{2}c_1(-1)^1(2-1)^5$

= 32+2(-1)

= 32 -2

= 30

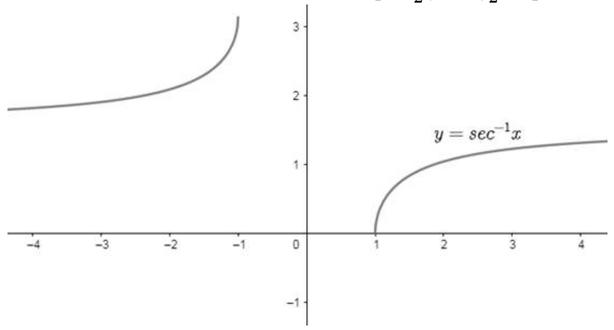
Hence Assertion is true.

The reason is false as the number of onto functions = $\sum_{r=0}^{n-1} {}^{n}c_r(-1)^r(n-r)^m$

so Assertion is true and reason is false.

Section B

21. Principal value branch of $\sec^1 x$ is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ and its graph is shown below.



OR

We have, $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

$\cos\left[\cos^{-1}\left(-cos\frac{\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left(\frac{5\pi}{6} + \frac{\pi}{6}\right)\{\because \cos^{-1}\cos x = x,\ x \in [0, \pi]\}$

$= \cos\left(\frac{6\pi}{6}\right)$

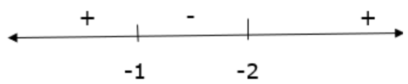
$= \cos(\pi) = -1$

22. Given, f(x) = $2x^3 + 9x^2 + 12x + 15$

f'(x) = $6x^2 + 18x + 12$

f'(x) = $6(x^2 + 3x + 2)$

= 6(x + 2)(x + 1)



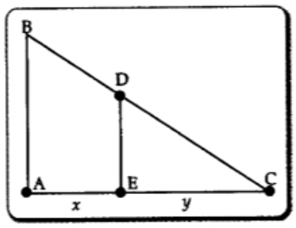
Therefore,function f(x) is decreasing for x ∈ [-1, -2] and increasing in x ∈ $(-\infty,-1)\cup(-2,\infty)$

23. Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC by y m.

Using similarity of triangles, we have

$\frac{4}{1.6}=\frac{x+y}{y}$

⇒ 3y = 2x



Differentiating both sides w.r.t. t, we get

$3\frac{dy}{dt}=2\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{2}{3}\times0.3\Rightarrow\frac{dy}{dt}=0.2$

At any time t seconds, the tip of his shadow is at a distance of (x + y) m from AB.

The rate at which the tip of his shadow moving $=\left(\frac{dx}{dt}+\frac{dy}{dt}\right)\mathrm{m/s}$ = 0.5 m/s

The rate at which his shadow is lengthening $=\frac{dy}{dt}\ \mathrm{m/s}=0.2\ \mathrm{m/s}$

OR

We have,

f(x) = $2x^3 - 3x^2 - 12x + 4$

f'(x) = $6x^2 - 6x - 12$

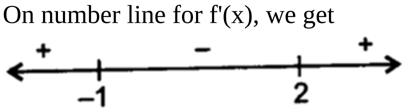
Now, f'(x) = 0

⇒ $6(x^2 - x - 2) = 0$

⇒ 6(x + 1)(x - 2) = 0

⇒ x = -1 and x = +2

On number line for f'(x), we get



Hence, x = -1 is point of local maxima and x = 2 is point of local minima.

24. Dividing numerator and denominator by $\cos^2x$ we have

$I=\int\frac{\sec^2x\,dx}{2\tan^2x+5}$

Put tan x = t so that $\sec^2x\,dx$ = dt. Then

$I=\int\frac{dt}{2t^2+5}$

$=\frac{1}{2}\int\frac{dt}{t^2+\left(\sqrt{\frac{5}{2}}\right)^2}$

$=\frac{1}{2}\left(\sqrt{\frac{2}{5}}\right)\tan^{-1}\left(\frac{t\sqrt{2}}{\sqrt{5}}\right)+C=\frac{1}{\sqrt{10}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{5}}\right)+C$

25. Since, we know that A(adj A) = $|A|^{n-1}$

Here, $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\therefore\ A.\,adj A=\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}^{2-1}\times I_2$

$=1^1\times I$

$=I$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

But $A(\text{adj } A)=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$ ...(given)

Hence, $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

⇒ k = 1

Hence, k = 1

Section C

26. We have,

$I = \int x^2 \tan^{-1} x dx$

considering $\tan^{-1}x$ as first function and x^2 as second function.. Then by using partial fraction we have.

$I = \tan^{-1} x \frac{x^3}{3} - \int \left(\frac{1}{1+x^2} \times \frac{x^3}{3}\right) dx$

$= \tan^{-1} x \frac{x^3}{3} - \frac{1}{3} \int \frac{x^3 dx}{1+x^2}$

$= \tan^{-1} x \frac{x^3}{3} - \frac{1}{3} \int \left(\frac{x^2 x}{1+x^2}\right) dx$

Putting $1 + x^2 = t$

⇒ $x^2 = t - 1$

⇒ 2x dx = dt

⇒ $x\mathrm{d}x = \frac{dt}{2}$

$\therefore I = \tan^{-1} x \frac{x^3}{3} - \frac{1}{6} \int \left(\frac{t-1}{t}\right) dt$

$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{6} \int dt + \frac{1}{6} \int \frac{dt}{t}$

$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{6}t + \frac{1}{6}\log|t| + C$

$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{6}(1 + x^2) + \frac{1}{6}\log|1 + x^2| + C$

$= \frac{x^3}{3}\tan^{-1} x - \frac{x^2}{6} + \frac{1}{6}\log|x^2 + 1| + C'$ where $C' = C - \frac{1}{6}$

27. When a die is thrown, probability of getting a six $(p) = \frac{1}{6}$

Therefore, q=probability of not getting a six = 1 - p $= 1 - \frac{1}{6} = \frac{5}{6}$

i. If he gets a six in first throw, then,

Probability of getting a six $= \frac{1}{6}$

ii. If he does not get a six, in first throw, but he gets a six in the second throw, then

Its probability $= \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

Probability that he does not get a six in first two throws and he gets a six in third throw

$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

Probability that he does not get a six in any of the three throws $= \left(\frac{5}{6}\right)^3 = \frac{125}{216}$

In first throw he gets a six, will receive Re.1

If he gets a six in second throw, he will receive Rs. (1 – 1) = 0

If he gets a six in third throw, he will receive Rs. (– 1 – 1 + 1) = Rs. – 1

= he will loss Rs. 1

If he does not get a six in all three throws, he will receive Rs. (–1 –1 –1) = Rs. –3

Expected value $= \frac{1}{6} \times 1 + \left(\frac{5}{6} \times \frac{1}{6}\right) \times 0 + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) \times (-1) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}\right) \times -3$

$= \frac{1}{6} - \frac{25}{216} - \frac{375}{216} = \frac{-364}{216} = \frac{-91}{54} = Rs.\ \frac{91}{54}\ Loss$

28. Given integral is: $\int_1^3 \frac{dx}{x^2(x+1)}$

To Prove: $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log \frac{2}{3}$

Let $I = \frac{dx}{(x^2)(x+1)}$

Using partial fraction:

Let $\frac{1}{(x^2)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$ .....(i)

⇒ $\frac{1}{(x^2)(x+1)} = \frac{A(x)(x+1)+B(x+1)+C(x^2)}{(x+1)(x^2)}$

$\Rightarrow 1 = \mathrm{A}\left(\mathrm{x}^2 + \mathrm{x}\right) + (\mathrm{Bx} + \mathrm{B}) + \mathrm{Cx}^2$

⇒ $1 = Ax^2 + Ax + B + Bx + Cx^2$

⇒ $1 = B + (A + B)x + (A + C)x^2$

Equating the coefficients of x, x^2 and constant value. We get:

B = 1

A + B = 0 ⇒ A = -B ⇒ A = -1

A + C = 0 ⇒ C = -A ⇒ C = 1

Put these values in equation (i)

$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}$

$\Rightarrow \int \frac{1}{(x^2)(x+1)} dx = \int -\frac{1}{x} dx + \int \frac{1}{(x^2)} dx + \int \frac{1}{(x+1)} dx$

$\Rightarrow \int_1^3 \frac{1}{(x^2)(x+1)} dx = \left[-\log|x| - x^{-1} + \log|x+1|\right]_1^3$

$\Rightarrow \int_1^3 \frac{1}{(x^2)(x+1)} dx = \left[-\frac{1}{x} + \log\left|\frac{x+1}{x}\right|\right]_1^3$

$= \left[-\frac{1}{3} + \log\left|\frac{3+1}{3}\right| - \left(-\frac{1}{1} + \log\left|\frac{1+1}{1}\right|\right)\right]$

$= \left[-\frac{1}{3} + \log\left|\frac{4}{3}\right| + \left(1 - \log\left|\frac{2}{1}\right|\right)\right]$

$= \left[-\frac{1}{3} + 1 + \log\left|\frac{4}{3} \times \frac{1}{2}\right|\right]$

$\Rightarrow \mathrm{I} = \left[\frac{2}{3} + \log\left|\frac{2}{3}\right]\right|$

L.H.S = R.H.S

Hence proved.

OR

Let the given integral be,

$\mathrm{I} = \int \frac{1}{\cos 2x + 3\sin^2 x} dx$

$= \int \frac{1}{(1-2\sin^2 x) + 3\sin^2 x} dx$

$= \int \frac{1}{1+\sin^2 x} dx$

Dividing numerator and denominator by $\cos^2 x$

$\Rightarrow I = \int \frac{\sec^2 x}{\sec^2 x + \tan^2 x} dx$

$= \int \frac{\sec^2 x}{1+\tan^2 x + \tan^2 x} dx$

$= \int \frac{\sec^2 x}{1+2\tan^2} dx$

$= \int \frac{\sec^2 x}{1+(\sqrt{2}\tan x)^2} dx$

Let $\sqrt{2}$ tan x = t

$\Rightarrow \sqrt{2}\sec^2 x\, dx = dt$

$\Rightarrow \sec^2 x\, dx = \frac{dt}{\sqrt{2}}$

$\therefore I = \frac{1}{\sqrt{2}} \int \frac{dt}{1+t^2}$

$= \frac{1}{\sqrt{2}} \tan^{-1}(t) + C$

$= \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2}\tan x) + C$

29. Given that, $(x + x^3)dy = -(1 + y + x^2y)dx$

$\Rightarrow \frac{dy}{dx} = \frac{-[1+y(1+x^2)]}{x(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = -\left[\frac{1}{x(1+x^2)} + \frac{y(1+x^2)}{x(1+x)^2}\right]$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{-1}{x(1+x^2)}$

$P = \frac{1}{x}, Q = \frac{-1}{x(1+x^2)}$

$I.F. = e^{\int p\, dx} = e^{\log x} = x$

$y \times x = \int \frac{-1}{x(1+x^2)} \times x\, dx + c$

$xy = -\tan^{-1} x + c$

OR

According to the question, $\frac{dy}{dx}$ + 2ytanx = sinx

Given equation is a linear differential equation in the form $\frac{dy}{dx}$ + Py = Q

Here, P = 2tanx and Q = sinx

Now, Integration Factor(IF) = $e^{\int Pdx} = e^{\int 2\tan x dx}$

$= e^{2\int \tan x dx}$

$= e^{2\log|\sec x|}$

$= e^{\log \sec^2 x}$

$= \sec^2 x \left[\because e^{\log f(x)} = f(x)\right]$

The solution of differential equation is given by

$y \cdot (IF) = \int (\text{IF}) Q dx + C$

$\Rightarrow y \cdot \sec^2 x = \int \sec^2 x \cdot \sin x dx + C$

$\Rightarrow y \cdot \sec^2 x = \int \frac{\sin x}{\cos^2 x} dx + C$

$\Rightarrow y \cdot \sec^2 x = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx + C$

$\Rightarrow y \cdot \sec^2 x = \int \tan x \cdot \sec x dx + C$

$\Rightarrow y \cdot \sec^2 x = \sec x + C$

Dividing with $\sec^2$x on both sides, we get

$\Rightarrow y = \frac{1}{\sec x} + \frac{C}{\sec^2 x}$

$\Rightarrow$y = cosx + C.$\cos^2$x

According to the question, y = 0 when x = $\frac{\pi}{3}$

$0 = \cos\frac{\pi}{3} + C \cdot \cos^2\frac{\pi}{3}$

$\Rightarrow 0 = \frac{1}{2} + C \cdot \frac{1}{4}$

$\Rightarrow \frac{-1}{2} = \frac{C}{4}$

$\Rightarrow C = -2$

Now, y = cosx + C.$\cos^2$x $\Rightarrow$ y = cos x - 2 $\cos^2$x

Therefore, the required particular solution is y = cos x - 2 $\cos^2$x

30. If $|\vec{a}| = 5$, $|\vec{b}| = 12$, $|\vec{c}| = 13$

If $(\vec{a} + \vec{b} + \vec{c}) = 0$.......(i)

Find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ = ?

Squaring the given equation (i) We get ,

$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

$\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c}$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

25 + 144 + 169 + 2(x) = 0

338 + 2x = 0

2x = -338

x = -169

hence , the required term is equal to -169.

OR

Let, $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$

Also, given $\vec{b} = 3\hat{i} + \hat{k}$

Also, let

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$ ... (i)

Where **u** is parallel to **b** and **v** is perpendicular to **b.**

now **u** is parallel to **b.**

$\vec{u} = \lambda\vec{b}$

$= \lambda(3\hat{i} + \hat{k})$

$\vec{u} = 3\lambda\hat{i} + \lambda\vec{k}$.... (ii)

put value of u in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = (3\lambda\hat{i} + \lambda\hat{k}) + \vec{v}$

$\vec{v} = 5\hat{j} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$

$\vec{v} = (5-3\lambda)\hat{i} - 2\hat{j} + (5-\lambda)\hat{k}$

Since, **v** is perpendicular to **b**

Then v.b =0

$[(5-3\lambda)\hat{i} + (-2)\hat{j} + (5-\lambda)\hat{k}] \cdot (3\hat{i} + 0 \times \hat{j} + \tilde{k}) = 0$

(5 - 3λ)(3) + (-2)(0) + (5 - λ)(1) = 0

$15 - 9\lambda + 0 + 5 - \lambda = 0$

$20 - 10\lambda = 0$

= > -10λ = -20

=> λ= 2

putting value of λ in equation (ii)

$\vec{u} = 3\lambda\hat{i} + \lambda\vec{k}$

$= 3(2)\hat{i} + (2)\hat{k}$

$\vec{u} = 6\hat{i} + 2\vec{k}$

put the value of u in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = (6\hat{i} + 2\hat{k}) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$

$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a} = (6\hat{i} + 2\vec{k}) + (-\hat{i} - 2\hat{j} + 3\hat{k})$

31. According to the question,if x = $\tan\left(\frac{1}{a}\log y\right)$, then we have to show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

We shall use product rule of differentiation to prove the above result.

Now, x = $\tan\left(\frac{1}{a}\log y\right)$

$\Rightarrow \tan^{-1}x = \frac{1}{a}\log y$

$\Rightarrow$ a tan ^{-1}x = log y

On differentiating both sides w.r.t x, we get,

$a \times \frac{1}{1+x^2} = \frac{1}{y} \cdot \frac{dy}{dx}$

$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = ay$

Again, differentiating both sides w.r.t x, we get,

$\left(1+x^2\right) \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx} \cdot \frac{d}{dx}\left(1+x^2\right) = \frac{d}{dx}(ay)$ [By using product rule of derivative]

$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot (2x) = a \cdot \frac{dy}{dx}$

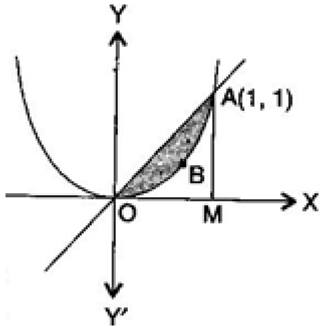
$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - a\frac{dy}{dx} = 0$

$\therefore \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

Hence Proved.

Section D

32. Equation of one curve (straight line) is y = x …..(i)



Equation of second curve (parabola) is y = x^2 ... (ii)

Solving eq. (i) and (ii), we get x = 0 or x = 1 and y = 0 or y = 1

$\therefore$ Points of intersection of line (i) and parabola (ii) are O (0, 0) and A (1, 1).

Now Area of triangle OAM

= Area bounded by line (i) and x - axis

$= \left|\int_0^1 y dx\right| = \left|\int_0^1 x dx\right| = \left(\frac{x^2}{2}\right)_0^1$

$= \frac{1}{2} - 0 = \frac{1}{2}$ sq units

Also Area OBAM = Area bounded by parabola (ii) and x - axis

$= \left|\int_0^1 y dx\right| = \left|\int_0^1 x^2 dx\right| = \left(\frac{x^3}{3}\right)_0^1$

$= \frac{1}{3} - 0 = \frac{1}{3}$ sq. units

$\therefore$ Required area OBA between line (i) and parabola (ii)

= Area of triangle OAM – Area of OBAM

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ sq. units

33. We have, $A = \{x \in Z : 0 \leq x \leq 12\}$ be a set and

R = {(a, b) : a = b} be a relation on A

Now,

Reflexivity: Let a ∈ A

⇒ a = a

⇒ (a, a) ∈ R

⇒ R is reflexive

Symmetric: Let a, b, ∈ A and (a, b) ∈ R

⇒ a = b

⇒ b = a

⇒ (b, a) ∈ R

⇒ R is symmetric

Transitive: Let a, b & c ∈ A

and let (a, b) ∈ R and (b, c) ∈ R

⇒ a = b and b = c

⇒ a = c

⇒ (a, c) ∈ R

⇒ R is transitive

Since R is being reflexive, symmetric and transitive, so R is an equivalence relation.

Also we need to find the set of all elements related to 1.

Since the relation is given by, R = {(a, b): a = b}, and 1 is an element of A.

R = {(1, 1): 1 = 1}

Thus, the set of all elements related to 1 is {1}.

OR

Given that,

R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}

Domain = {1,2,3,.......,20}

Range = {1,3,5,7......,39}

R is not reflexive as (2, 2) ∉ R as

$2 \times 2 + 2 \neq 41$

R is not symmetric

as (1, 39) ∈ R but (39, 1) ∉ R

R is not transitive

as (11, 19) ∈ R, (19, 3) ∈ R

But (11, 3) ∉ R

Hence, R is neither reflexive, nor symmetric and nor transitive.

34. L.H.S. $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

Now, $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

R.H.S. $= (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

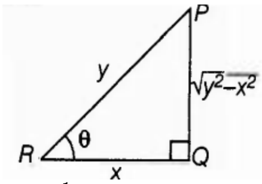
$$= \begin{bmatrix} \cos\alpha + \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & -\sin\alpha + \cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ -\cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \sin\alpha & \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\alpha\cos\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\alpha\cos\frac{\alpha}{2} + \cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{-\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\sin\alpha\sin\frac{\alpha}{2} + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{-\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \\ \frac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$\therefore$ L.H.S. = R.H.S. Proved.

35. Let us consider a right-angled triangle with base = x and hypotenuse = y. Let x + y = k, where k is a constant. Let θ be the angle between the base and the hypotenuse. Let A be the area of the triangle, then



$A = \frac{1}{2} \times QR \times PQ = \frac{1}{2}x\sqrt{y^2 - x^2}$

$\Rightarrow \quad A^2 = \frac{x^2\left(y^2 - x^2\right)}{4}$

$\Rightarrow \quad A^2 = \frac{x^2\left[(k-x)^2 - x^2\right]}{4}$ $[\because y = k - x]$

$\Rightarrow \quad A^2 = \frac{k^2x^2 - 2kx^3}{4}$ .....(i)

$\Rightarrow \quad 2A\frac{dA}{dx} = \frac{2k^2x - 6kx^2}{4}$ ...(ii)

$\Rightarrow \quad \frac{dA}{dx} = \frac{k^2x - 3kx^2}{4A}$

Now, for maxima or minima

Put $\frac{dA}{dx} = 0 \Rightarrow \left(k^2x - 3kx^2\right) = 0 \Rightarrow x = \frac{k}{3}$

Differentiating (ii) w.r.t x,we get,

$2\left(\frac{dA}{dx}\right)^2 + 2A\frac{d^2A}{dx^2} = \frac{2k^2 - 12kx}{4}$ ....(iii)

Put $\frac{dA}{dx} = 0$ and $x = \frac{k}{3}$ in Eq. (iii), we get

$\frac{d^2A}{dx^2} = \frac{-k^2}{4A} < 0$

Thus, A is maximum when $x = \left(\frac{k}{3}\right)$

Now, $x = \frac{k}{3}$

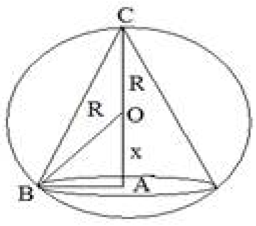
$\Rightarrow \quad y = \left(k - \frac{k}{3}\right) = \frac{2k}{3}$

$\therefore \quad \frac{x}{y} = \cos\theta$

$\Rightarrow \quad \cos\theta = \frac{k/3}{2k/3} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Hence, the area of triangle is maximum, when $\theta = \frac{\pi}{3}$.

OR



$v = \frac{1}{3}\pi r^2 h \ \left[r^2 = \sqrt{R^2 - x^2}\right]$

$V=\frac{1}{2}\pi.\left(R^2-x^2\right).(R+x)$

$\frac{dy}{dx}=\frac{1}{3}\pi\left[\left(R^2-x^2\right)(1)+(R+x)(-2x)\right]$

$=\frac{1}{3}\pi\left[(R+x)(R-x)-2x(R+x)\right]$

$=\frac{1}{3}\pi(R+x)[R-x-2x]$

$=\frac{1}{3}\pi(R+x)(R-3x)$....(1)

Put $\frac{dv}{dr}=0$

R = - x (neglecting)

R = 3x

$\frac{R}{3}=x$

On again differentiating equation (1)

$\frac{d^2v}{dx^2}=\frac{1}{3}\pi[(R+x)(-3)+(R-3x)(1)]$

$=\frac{d^2v}{dx^2}\Big]_{x=\frac{R}{3}}=\frac{1}{3}\pi\left[\left(R+\frac{R}{3}\right)(-3)+\left(R-3.\frac{R}{3}\right)\right]$

$\frac{1}{3}\pi\left[\frac{4R}{3}\times-3+0\right]$

$=\frac{-1}{3}\pi 4R$

$\frac{d^2v}{dx^2}<0$ Hence maximum

Now $v=\frac{1}{3}\pi\left[\left(R^2-x^2\right)(R+x)\right]\left[x=\frac{R}{3}\right]$

$v=\frac{1}{3}\pi\left[\left(R^2-\left(\frac{R}{3}\right)^2\right)\left(R+\left(\frac{R}{3}\right)\right)\right]$

$=\frac{1}{3}\pi\left[\frac{8R^2}{9}\times\frac{4R}{3}\right]$

$v=\frac{8}{27}\left(\frac{4}{3}\right)\pi R^3$

$v=\frac{8}{27}$ Volume of sphere

Volume of cone $=\frac{8}{27}$ of volume of sphere.

Section E

36. i. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

= $\frac{P(E\cap M)}{P(M)}=\frac{\frac{1}{4}}{\frac{7}{20}}$ = $\frac{1}{4}\times\frac{20}{7}=\frac{5}{7}$

ii. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$=\frac{P(M\cap E)}{P(E)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$

iii. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P(M'/E)=\frac{P(M'\cap E)}{P(E)}$

= $\frac{P(E)-P(E\cap M)}{P(E)}$

= $\frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P(M'/E)=\frac{1}{2}$

OR

Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in

Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P(E'/M)=\frac{P(E'\cap M)}{P(M)}$

= $\frac{P(M)-P(E\cap M)}{P(M)}$

= $\frac{\frac{7}{20}-\frac{1}{4}}{\frac{7}{20}}\Rightarrow P(E'/M)=\frac{2}{7}$

37. i. Clearly, the coordinates of A are (8, 10, 0) and D are (0, 0, 30)

$\therefore$ Equation of AD is given by

$\frac{x-0}{8-0}=\frac{y-0}{10-0}=\frac{z-30}{-30}$

$\Rightarrow\frac{x}{4}=\frac{y}{5}=\frac{30-z}{15}$

ii. The coordinates of point C are (15, - 20, 0) and D are (0, 0, 30)

$\therefore$ Length of the cable DC

$=\sqrt{(0-15)^2+(0+20)^2+(30-0)^2}$

$=\sqrt{225+400+900}=\sqrt{1525}=5\sqrt{61}$ m

iii. Since, the coordinates of point B are (-6, 4, 0) and D are (0, 0, 30), therefore vector DB is

$(-6-0)\hat{i}+(4-0)\hat{j}+(0-30)\hat{k}$, i.e., $-6\hat{i}+4\hat{j}-30\hat{k}$

OR

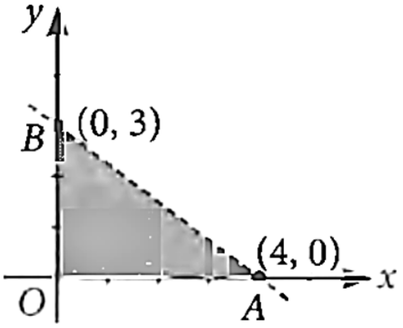
Required sum

= $(8\hat{i}+10\hat{j}-30\hat{k})+(-6\hat{i}+4\hat{j}-30\hat{k})+(15\hat{i}-20\hat{j}-30\hat{k})$

= $17\hat{i}-6\hat{j}-90\hat{k}$

38. i. When we solve an L.P.P. graphically, the optimal (or optimum) value of the objective function is attained at corner points of the feasible region.

ii. From the graph of 3x + 4y < 12 it is clear that it contains the origin but not the points on the line 3x + 4y = 12.



iii. Maximum of objective function occurs at corner points.

Corner Points	Value of Z = 2x + 5y
(0, 0)	0
(7, 0)	14
(6, 3)	27
(4, 5)	33 ← Maximum
(0, 6)	30

OR

Value of Z = px + qy at (15, 15) = 15p + 15q and that at (0, 20) = 20q. According to given condition, we have

15p + 15q = 20q ⇒ 15p = 5q ⇒ q = 3p